

Critical Behavior and Critical Exponents of Rotating QCD Matter

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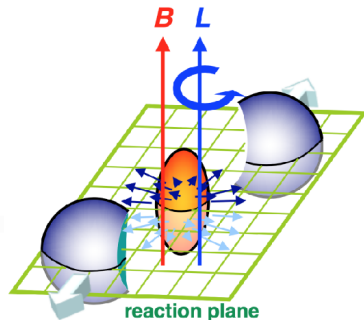
The 2nd Workshop on QCD Matter under Rotation and Magnetic Fields, Huainan, Anhui

Why rotation matters for QCD matter

- Non-central heavy-ion collisions generate large orbital angular momentum and substantial vorticity.
- Spin-orbit coupling transfers part of the orbital angular momentum to spin degrees of freedom.
- Global hyperon polarization and vector-meson spin alignment motivate the study of rotating QCD matter.
- Rotation can modify chiral and deconfinement transitions and therefore enlarge the thermodynamic control space.

Central idea

Angular velocity ω is a thermodynamic control parameter conjugate to angular momentum density J .

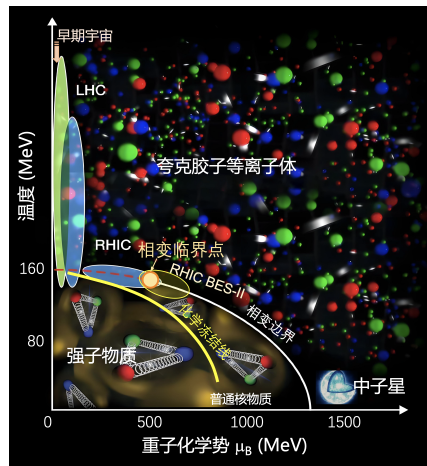


Maxim Chernodub

Critical endpoint

- A CEP terminates a first-order transition line and connects it to a crossover region.
- Near the CEP, response functions become singular and scaling laws emerge.
- Standard critical exponents: $\alpha, \beta, \gamma, \delta$.
- In rotating matter, the relevant parameter space is (T, ω) .

Question: How is the conventional critical scaling structure manifested through rotational observables?



Two-flavor NJL model in a rotating frame

For uniform rotation around the z-axis, $\boldsymbol{\omega} = \omega \hat{z}$, the model is

$$\begin{aligned}\mathcal{L} = & \bar{\psi} \left(i\gamma^\mu \partial_\mu - \hat{m} + \gamma^0 \mu \right) \psi \\ & + \bar{\psi} [(\boldsymbol{\omega} \times \mathbf{x}) \cdot (-i\nabla) + \boldsymbol{\omega} \cdot \mathbf{S}] \psi \\ & + G \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5 \boldsymbol{\tau}\psi)^2 \right].\end{aligned}$$

- Isospin symmetry: $m_u = m_d = m$, $\hat{m} = m\mathbf{1}_f$.
- Mean field: $\langle \bar{\psi}\psi \rangle \neq 0$, pseudoscalar condensate set to zero.
- Rotation couples to both orbital and spin angular momenta.

Order-parameter-like variable

$M = m - 2G\langle \bar{\psi}\psi \rangle$ is used to track chiral symmetry breaking/restoration.

Local grand potential and equilibrium condition

At vanishing quark chemical potential,

$$\Omega = \frac{(M - m)^2}{4G} - \frac{N_f N_c}{2\pi^2} \sum_n \int_0^\Lambda k_t dk_t \int_{-\sqrt{\Lambda^2 - k_t^2}}^{\sqrt{\Lambda^2 - k_t^2}} dk_z \\ \times \mathcal{J}_n(k_t r) T \ln \left[2 \cosh \left(\frac{\varepsilon_n}{2T} \right) \right],$$

with

$$\varepsilon_n = E_k - \left(n + \frac{1}{2} \right) \omega, \quad E_k = \sqrt{k_t^2 + k_z^2 + M^2}, \quad \mathcal{J}_n = J_n^2 + J_{n+1}^2.$$

Equilibrium

$$\frac{\partial \Omega}{\partial M} = 0, \quad \frac{\partial^2 \Omega}{\partial M^2} > 0.$$

Interpretation

Because rotation breaks transverse translational invariance, Ω is treated as a **local grand-potential density** at fixed r .

Rotating thermodynamics: the conjugate pair (ω, J)

$$d\varepsilon = T ds + \mu dn + \omega dJ,$$

$$\Omega = \varepsilon - Ts - \mu n - \omega J,$$

$$d\Omega = -s dT - n d\mu - J d\omega.$$

Therefore,

$$s = - \left(\frac{\partial \Omega}{\partial T} \right)_{\mu, \omega}, \quad n = - \left(\frac{\partial \Omega}{\partial \mu} \right)_{T, \omega}, \quad \boxed{J = - \left(\frac{\partial \Omega}{\partial \omega} \right)_{T, \mu}}.$$

Thermodynamic analogy

$$\mu \leftrightarrow \omega, \quad n \leftrightarrow J.$$

Angular velocity is the rotational control variable; J is its conjugate response.

Four effective exponents and the approach trajectories

Exponent	Scaling law	Operational trajectory / observable
α_ω	$C_J \sim t ^{-\alpha_\omega}$	vary T , evaluate along $\omega = \omega_{\text{CEP}}$
β_ω	$\Delta J \sim t ^{\beta_\omega}$	approach CEP along first-order coexistence line
γ_ω	$\chi_\omega \sim t ^{-\gamma_\omega}$	trajectory tangent to the phase boundary near CEP
δ_ω	$ \tilde{J} \sim \tilde{\omega} ^{1/\delta_\omega}$	critical isotherm $T = T_{\text{CEP}}$

$$t = \frac{T - T_{\text{CEP}}}{T_{\text{CEP}}}, \quad \tilde{J} = J - J_{\text{CEP}}, \quad \tilde{\omega} = \frac{\omega - \omega_{\text{CEP}}}{\omega_{\text{CEP}}}.$$

Interpretation

These are **effective, path-defined exponents**: physical (T, ω) coordinates need not coincide exactly with asymptotic thermal- and field-like scaling fields.

Thermal response and α_ω

The specific heat density at fixed rotational polarization is

$$C_J = T \left(\frac{\partial s}{\partial T} \right)_J$$

Using $s = -\Omega_T$ and $J = -\Omega_\omega$,

$$C_J = -T \left[\Omega_{TT} - \frac{\Omega_{T\omega}^2}{\Omega_{\omega\omega}} \right]$$

where Ω denotes the equilibrium-reduced grand potential. Critical scaling:

$$C_J \sim |t|^{-\alpha_\omega}, \quad \alpha_\omega(t_i) = -\frac{\Delta \ln C_J}{\Delta \ln |t|}.$$

Local-slope procedure

Local-slope procedure avoids the need to select a global fitting window and allows the evolution of the effective exponent to be monitored as the CEP is approached.

Coexistence discontinuity and β_ω

$$J = -\frac{\partial \Omega}{\partial \omega},$$

$$\Delta J = J_>(T, \omega, M_>) - J_<(T, \omega, M_<),$$

$$\boxed{\Delta J \sim |t|^{\beta_\omega}}.$$

- On the first-order line, two thermodynamically degenerate minima coexist.
- They have different constituent masses $M_>$ and $M_<$, and hence different J .
- As the CEP is approached, the two phases merge and $\Delta J \rightarrow 0$.

Important qualification

J is not the conventional chiral order parameter; its singular part has a nonzero projection onto the CEP critical mode.

Rotational susceptibility and γ_ω

$$\chi_\omega = \frac{\partial J}{\partial \omega}$$

Because M is determined self-consistently,

$$\chi_\omega = -\Omega_{\omega\omega} + \frac{\Omega_{\omega M}^2}{\Omega_{MM}}$$

- first term: direct response to ω ,
- second term: response mediated by the soft chiral mode,
- at the CEP: $\Omega_{MM} \rightarrow 0$, enhancing χ_ω .

Scaling

$$\chi_\omega \sim |t|^{-\gamma_\omega}$$

along a trajectory tangent to the phase boundary.

Critical-isotherm response and δ_ω

Near the CEP,

$$J - J_{\text{CEP}} \simeq -\Omega_{\omega M}^{\text{CEP}} (M - M_{\text{CEP}}) - \Omega_{\omega\omega}^{\text{CEP}} (\omega - \omega_{\text{CEP}}) + \dots$$

If $\Omega_{\omega M}^{\text{CEP}} \neq 0$, then the singular part of J inherits the leading scaling of the critical mode.

$$\tilde{J} = J - J_{\text{CEP}}, \quad \tilde{\omega} = \frac{\omega - \omega_{\text{CEP}}}{\omega_{\text{CEP}}},$$

$$|\tilde{J}| \sim |\tilde{\omega}|^{1/\delta_\omega}, \quad T = T_{\text{CEP}}.$$

Physical meaning

δ_ω quantifies the **nonlinear rotational response** on the critical isotherm.

Numerical setup and interpretation

- $m = 0.005 \text{ GeV}$
- $G = 3.672 \text{ GeV}^{-2}$
- $\Lambda = 0.6816 \text{ GeV}$
- fixed radial coordinate: $r = 0.1 \text{ GeV}^{-1}$
- angular-momentum modes: $-5 \leq n \leq 5$

Local causality

At the CEP,

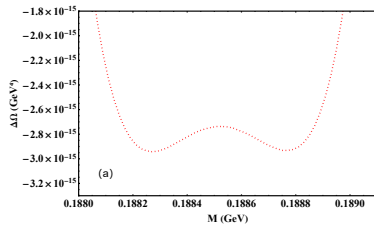
$$\omega_{\text{CEP}} r \simeq 0.0644 < 1.$$

Scope

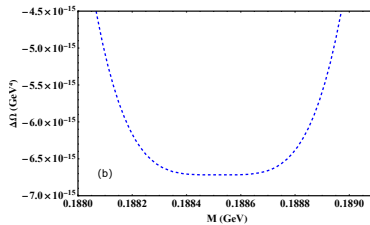
Local thermodynamic analysis at fixed r ;
not a fully volume-averaged finite rotating
cylinder.

CEP values are rounded for presentation; higher-precision values are retained internally for near-critical scaling.

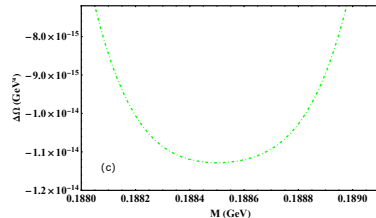
Thermodynamic-potential landscape and CEP location



First-order line: two degenerate minima



CEP: merged, flattened minimum



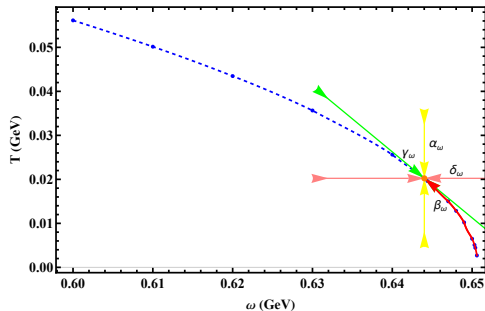
Crossover: single minimum

Numerical CEP

$$T_{\text{CEP}} \simeq 0.02023 \text{ GeV}, \quad \omega_{\text{CEP}} \simeq 0.6440 \text{ GeV}$$

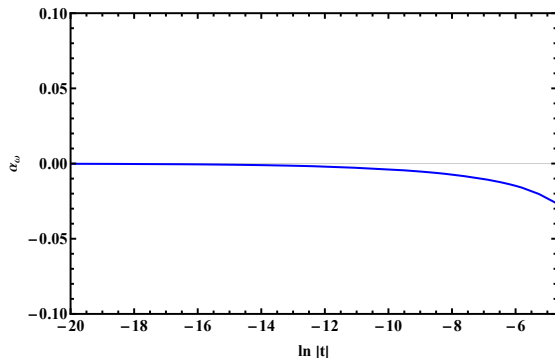
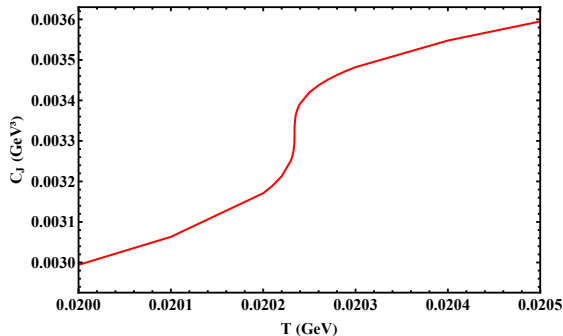
At the CEP, $\Delta M \rightarrow 0$ and $\Omega_{MM} \rightarrow 0$.

Schematic of the QCD phase diagram and the paths of exponents approaching the CEP



Exponent	Observable	Physical content
α_ω	C_J	thermal response
β_ω	ΔJ	vanishing coexistence discontinuity
γ_ω	χ_ω	divergent rotational susceptibility
δ_ω	J	nonlinear critical-isotherm response

Result I: thermal response and α_ω

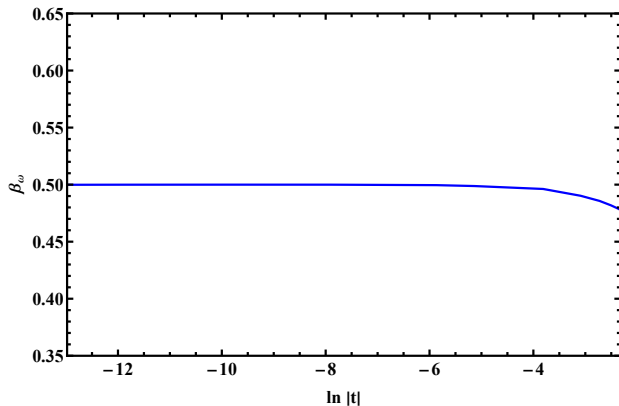


Result

$$\alpha_\omega \simeq 0$$

No power-law divergence of C_J within mean field; behavior is consistent with a finite discontinuity.

Result II: coexistence scaling and β_ω



Scaling law

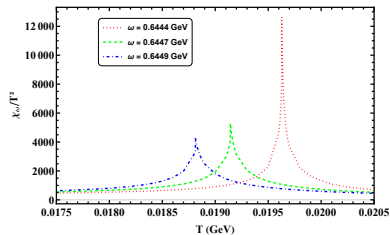
$$\Delta J \sim |t|^{\beta_\omega}$$

Result

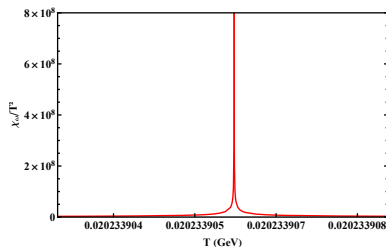
$$\beta_\omega \simeq \frac{1}{2}$$

- plateau near the CEP,
- deviations farther away: preasymptotic regime.

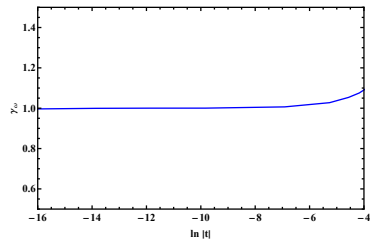
Result III: rotational susceptibility and γ_ω



susceptibility sharpens as $\omega \rightarrow \omega_{\text{CEP}}$



sharply enhanced response at $\omega = \omega_{\text{CEP}}$

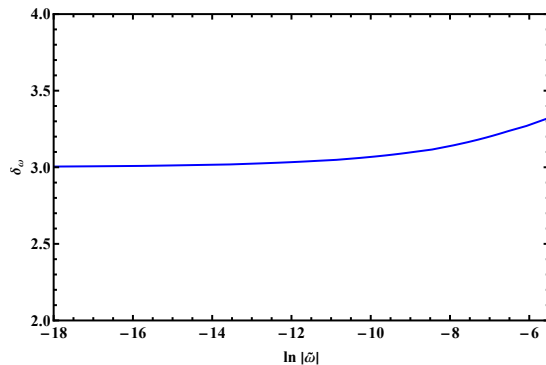
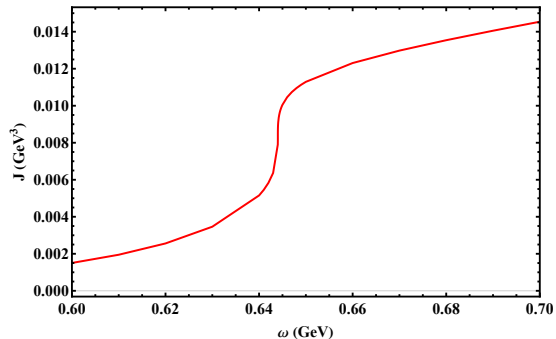


Result

$$\gamma_\omega \simeq 1$$

The divergence is tied to the softening of the chiral curvature $\Omega_{MM} \rightarrow 0$.

Result IV: critical isotherm and δ_ω



Result

$$\delta_\omega \simeq 3$$

Physically, δ_ω quantifies how strongly the rotational polarization responds nonlinearly to an infinitesimal change in angular velocity exactly at the CEP.

A coherent mean-field scaling picture

$$\alpha_\omega \simeq 0, \quad \beta_\omega \simeq \frac{1}{2}, \quad \gamma_\omega \simeq 1, \quad \delta_\omega \simeq 3$$

Scaling checks

$$\begin{aligned}\alpha_\omega + 2\beta_\omega + \gamma_\omega &= 2, \\ \alpha_\omega + \beta_\omega(1 + \delta_\omega) &= 2.\end{aligned}$$

Physical message

Rotation changes the thermodynamic control space and the phase structure, while the extracted asymptotic scaling remains consistent with Landau mean-field behavior in this NJL treatment.

Novelty: the scaling structure is demonstrated through mutually consistent thermal and rotational response observables, not a single fit.

Conclusions and outlook

Messages

- The conjugate pair (ω, J) provides a natural framework for rotational critical responses.
- Four complementary probes yield a mutually consistent mean-field exponent set.
- The results provide a benchmark for extensions beyond the minimal mean-field setup.

Next steps

Finite-volume/boundary effects; vector interactions; PNJL/deconfinement dynamics; beyond-mean-field fluctuations and universality-class analysis.

Thank you for your attention.

Questions and discussion